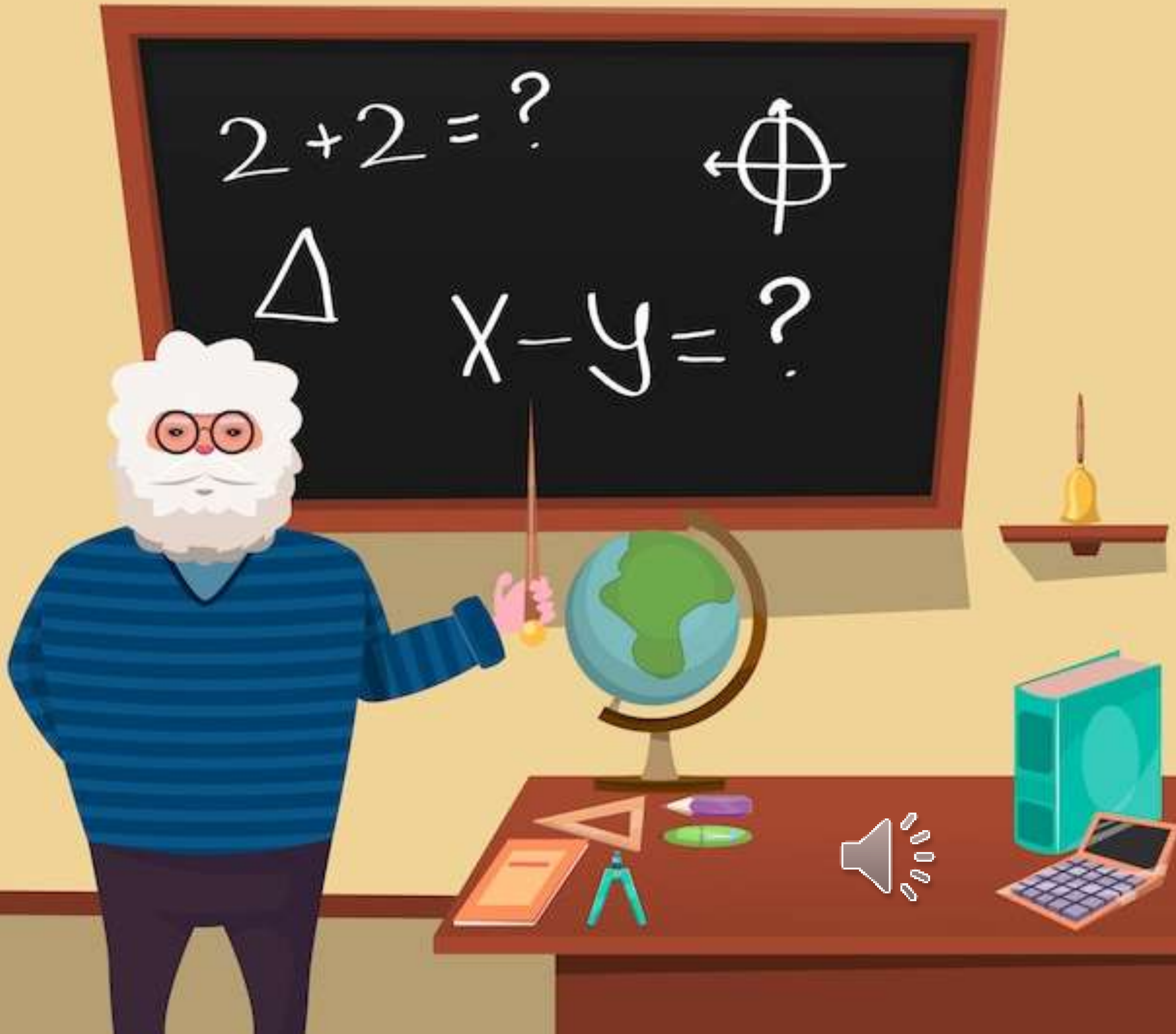
Systems of equations



Equation of first degree in two variables

General form: $ux + vy + w = 0$

Example: $2x - y + 1 = 0$

Number of solutions: Infinity of solutions

Example: $2x - y + 1 = 0$

(0;1) is a solution since 2(0)-1+1=0 so 0=0

(1;3) is a solution since 2(1)-3+1=0 so 0=0

(-1;-1) is a solution since 2(-1)-(-1)+1=0 so 0=0

(2;5) is a solution since 2(2)-5+1=0 so 0=0

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Remark:

(0;1), (1;3), (-1;-1), … are called ordered pairs where the first number represent the value of x and the second the value of y.

Equation of first degree in two variables

Graphical representation: Straight line

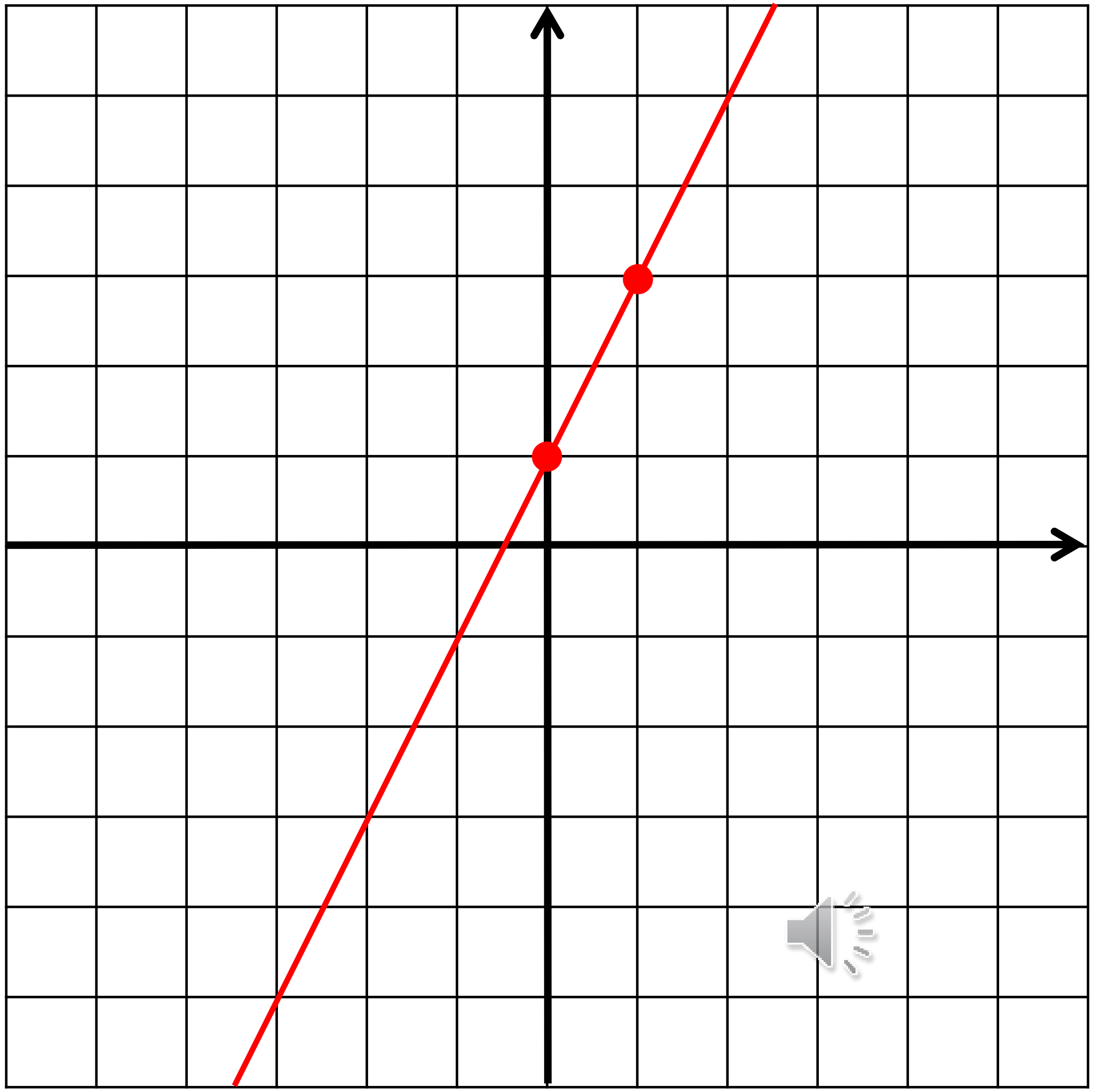
Example: $2x - y + 1 = 0$

$2x + 1 = y$

So $y = 2x + 1$

2 points are needed:

- For $x = 0$; $y = 2(0) + 1 = 1$
- For $x = 1$; $y = 2(1) + 1 = 3$



System of equations of the first degree in two variables

- Two equations of the first degree in two variables form a system of two equations.

Example: $\begin{cases} 2x + 3y = -1 \\ x + y = 1 \end{cases}$ is a system of two equations in two variables.

- Solving the system:

find the solution that verifies the two equations simultaneously.

Example: the above system has (4;-3) as a solution since:

$2(4) + 3(-3) = 8 - 9 = -1$

And

$4 + (-3) = 4 - 3 = 1$

System of equations of the first degree in two variables

Application # 1

Consider the following system of two equations in two variables:

$$(S)\begin{cases} 3x - 5y = 8 \\ 2x - 5y = 7 \end{cases}$$

Does (1;-1) a solution of (S)?

3(1)-5(-1)=3+5=8 so (1;-1) verifies the first equation

$2(1) - 5(-1) = 2 + 5 = 7$ so (1;-1) verifies the second equation

So (1;-1) is a solution of (S).

System of equations of the first degree in two variables

Application # 2

Consider the following system of two equations in two variables:

$$(S)\begin{cases} x - y = 3 \\ 2x + y = 2 \end{cases}$$

Does (-1;-4) a solution of (S)?

-1-(-4)=-1+4=3 so (-1;-4) verifies the first equation

2(-1)+(-4)=-2-4=-6≠2 so (-1;-4) doesn’t verify the second equation.

Hence (-1;-4) is not a solution of (S).

Solving a system of two equations

Method ❶: Graphically

Consider the system:

$$(S)\begin{cases} x - y = 3 \\ 2x + y = 2 \end{cases}$$

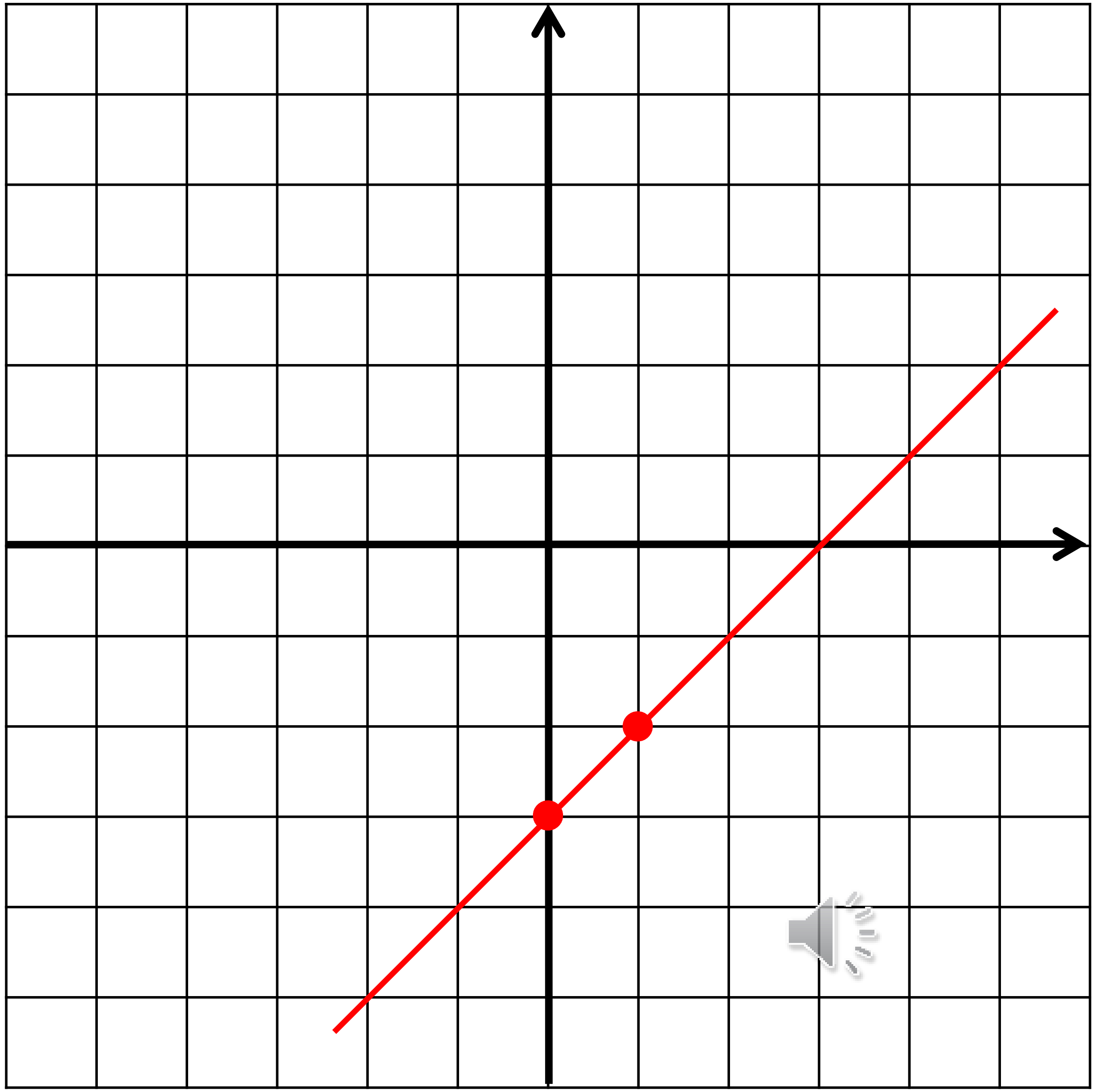
Each equation represent a line.

Equation 1

$x - y = 3$; $y = x - 3$

For $x = 0$; $y = 0 - 3 = -3$

For $x = 1$; $y = 1 - 3 = -2$



Solving a system of two equations

Method ❶: Graphically

Consider the system:

$$(S)\begin{cases} x - y = 3 \\ 2x + y = 2 \end{cases}$$

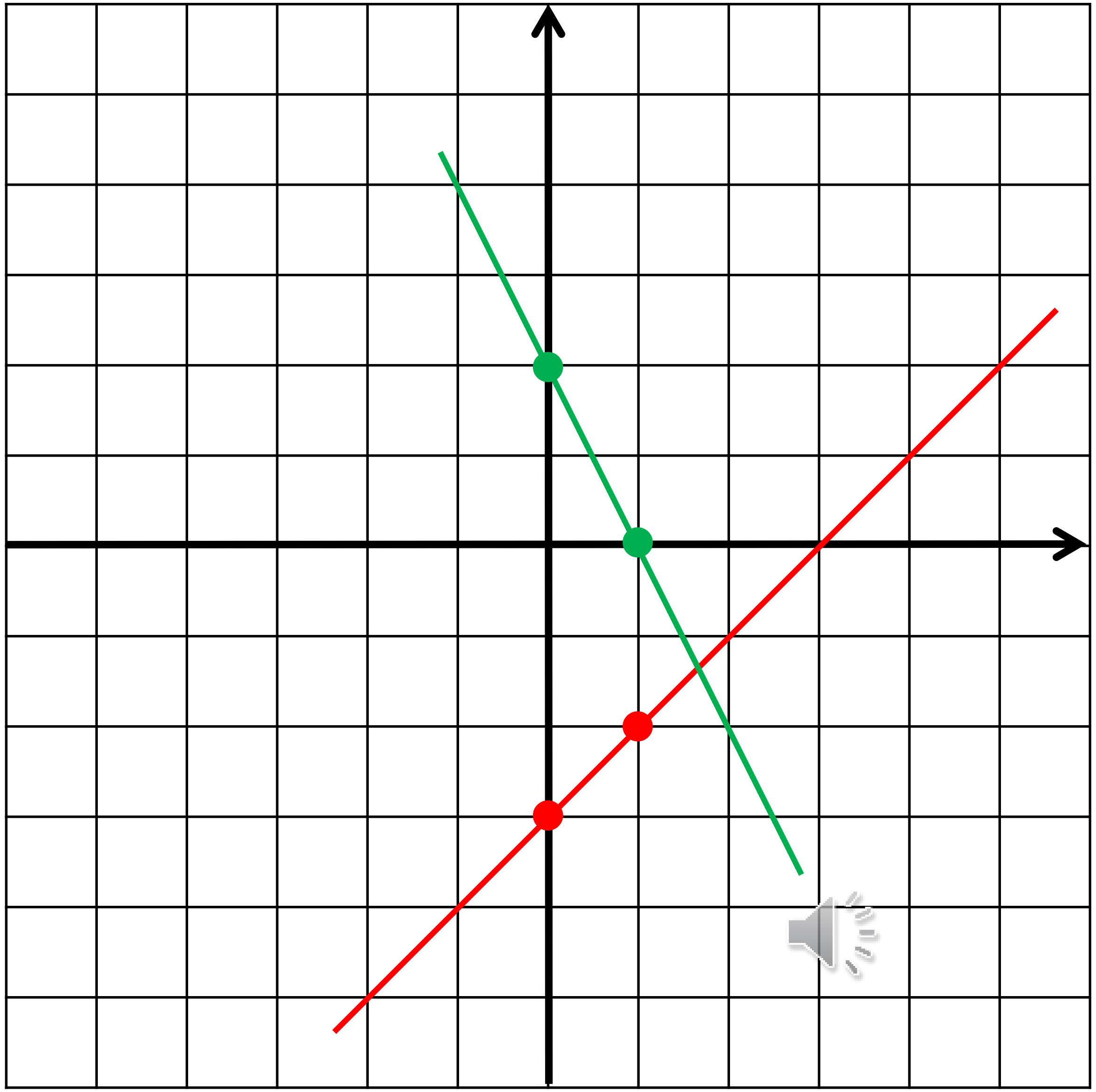
Each equation represent a line.

Equation 2

$2x + y = 2$; $y = -2x + 2$

For $x = 0$; $y = 0 + 2 = 2$

For $x = 1$; $y = -2 + 2 = 0$



Solving a system of two equations

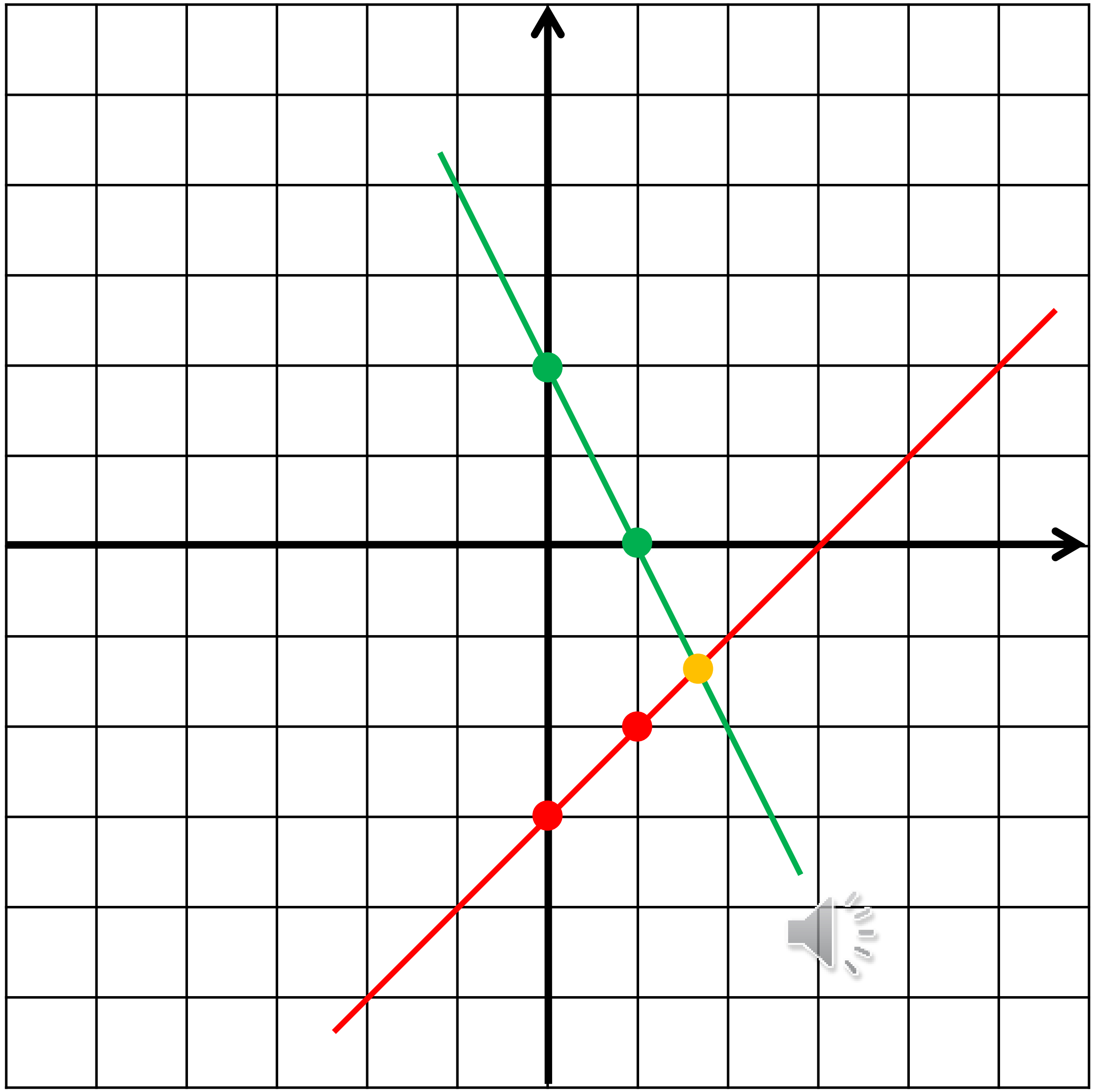
Method ❶: Graphically

Consider the system:

$$(S)\begin{cases} x - y = 3 \\ 2x + y = 2 \end{cases}$$

Each equation represent a line.

The solution is the intersecting point of the two lines



Solving a system of two equations

Method ❷: Algebraic methods

By elimination

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Elimination method consists to eliminate one of the two variables x or y by changing the coefficient of one of them to be opposite

Step ❶: Choose the variable to eliminate based on the coefficients

$$(S)\begin{cases} 1x + 1y = 1 \\ 2x + 3y = -1 \end{cases}$$

The coefficient of both x and y are easy to change

Choose any of the two variables: example x

Solving a system of two equations

Method ❷: Algebraic methods

By elimination

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Step ❷: change the coefficients of x to be opposite by multiplication

$$\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases} \begin{array}{c} \xrightarrow{\times(2)} \\ \xrightarrow{\times(-1)} \end{array} \begin{cases} 2x + 2y = 2 \\ -2x - 3y = 1 \end{cases}$$

Solving a system of two equations

Method ❷: Algebraic methods

By elimination

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Step ❸: Add the two equations to have an equation in one variable, then solve it.

$$\begin{cases} 2x + 2y = 2 \\ -2x - 3y = 1 \end{cases} \quad +$$

$$2y - 3y = 2 + 1$$

$$-y = 3$$

$$y = -3$$

Solving a system of two equations

Method ❷: Algebraic methods

By elimination

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Step ❹: Substitute the obtained value of y in one of the two main equations.

$$\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

$$y = -3$$

$$x + y = 1$$

$$x + (-3) = 1$$

$$x = 1 + 3 = 4$$

So the solution is (-2;4)

Solving a system of two equations

Method ❷: Algebraic methods

By substitution

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Step ❶: choose one of the two equations and write x in terms of y or y in terms of x.

$$x + y = 1$$

$$x = 1 - y$$

Solving a system of two equations

Method ❷: Algebraic methods

By substitution

Step ❷: substitute in the second equation.

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

$x = 1 - y$

$2(1 - y) + 3y = -1$

$2 - 2y + 3y = -1$

$2 + y = -1$

$y = -1 - 2 = -3$

Solving a system of two equations

Method ❷: Algebraic methods

By substitution

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ 2x + 3y = -1 \end{cases}$$

Step ❸: find the value of the second variable

$y = -3$

$x = 1 - y = 1 - (-3) = 1 + 3 = 4$

So (4;-3) is the solution

Solving a system of two equations

Method ❷: Algebraic methods

By comparison

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ x + 3y = -1 \end{cases}$$

Step ❶: write on e f the two variables in terms of the second variable in the two equations

$x + y = 1 \quad ; \quad x = 1 - y$

$x + 3y = -1 \; ; \; x = -1 - 3y$

Solving a system of two equations

Method ❷: Algebraic methods

By comparison

Step ❷: solve by comparing the two expressions

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ x + 3y = -1 \end{cases}$$

$x = 1 - y$

$x = -1 - 3y$

$x = x$

$1 - y = -1 - 3y$

$-y + 3y = -1 - 1$

$2y = -2\ ;\ y = -1$

Solving a system of two equations

Method ❷: Algebraic methods

By comparison

Step ❸: Find the value of the other variable

Consider the system:

$$(S)\begin{cases} x + y = 1 \\ x + 3y = -1 \end{cases}$$

$$x = 1 - y = 1 - (-1) = 1 + 1 = 2$$